COSMIC RAY MULTIPLE MUON EVENTS IN DEEP DETECTORS

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A simple model is developed which predicts the average number of multi-TeV muons produced in cosmic ray air showers. The model also give the approximate rates at which different numbers of muons reach a detector's depth. The dependences of the rates on zenith angle, detector depth, primary spectrum and composition, and interaction properties are given. The results are briefly discussed in terms of DUMAND and the Homestake Neutrino Detector.

Introduction. Colliding beam accelerators can be expected to reach energies equivalent to laboratory energies above 10^{14} eV during the next decade. It will be possible to study the composition of cosmic rays in the energy region 3×10^{14} to 3×10^{16} eV by extrapolating the interaction characteristics over a smaller energy interval than has previously been necessary. In another paper submitted to this conference,[1] the sensitivity of several kinds of shower measurements to the primary composition is analyzed using a scaling model for the nuclear interactions. Although the model may not be correct in detail, the calculated dependence on the composition of the various quantities is useful as a guide for planning experiments.

This work was started in order to investigate the use of DUMAND as a deep muon detector and a more detailed discussion of some of the results presented in this paper is available.[2] For large detectors in which the detector dimensions are much larger than the typical separation of high energy muons, a simplified analysis is possible and will be used below to estimate the rates of multiple muon events in the detector. Direct muon production, expected to become important in this energy region, is not included in this calculation. As the understanding of direct muon production improves, it will be possible to incorporate it into the model.

If extremely large detectors such as DUMAND are constructed, it may be possible to detect very massive particles (10-100 GeV/c^2) if they sometimes yield a muon in their decay process. The resulting muon may receive a transverse momentum up to half the particle mass times c. This may be the only way by which extremely high transverse momenta are generated, and a massive particle might be detected by observation of widely separated energetic muons.

Model of Multiple Muon Events. This section describes a specific model for multiple muon production by the decay of pions and kaons. Ultimately, accurate Monte-Carlo studies will need to be done, but the model can give a rough estimate of event rates and the sensitivity of experiments to the primary composition, so that plans may be made for future experiments.

The interaction model used to obtain the average numbers of multiple muon events is described elsewhere.[1,3] It is based on Feynman scaling and accelerator measurements with nuclear targets. The cross sections for hadron-air interactions follow the energy dependence given by Gaisser and Yodh.[4] This dependence is in good agreement with recent accelerator results.

Since the muons are produced by meson decay in this model and the meson fluxes are calculated using a scaling model, it is not surprising that muon fluxes can be represented as a product of a scaling factor and a decay factor. Although the model does not obey scaling completely because of the energy dependent cross sections, the effect of this scaling violation is not important. The decay probability introduces a factor ($E_ì \cos \theta$)$^{-1}$ into the muon production, where Eu is the muon energy and a is the zenith angle of the shower. For $E_ì \cos \theta > 10^3$ GeV and $\theta < 80°$, the results of the calculation are given by

$$N_\mu(> E_\mu) = \frac{A\ G(x)}{E_\mu \cos\theta} \qquad (E_\mu \text{ in GeV}) \qquad (1)$$

The quantity $N_ì$ (> $E_ì$) is the number of muons at ground level with energy greater than $E_ì$. The variable A is the atomic mass of the incident nucleus (the superposition model is being used here) and x is $E_ì$ A/E_o, where E_o is the total shower energy. Thus x is the ratio of the muon energy to the energy/ nucleon of the incident nucleus. The function G(x) = $14.5\ x^{-0.757}$ $(1 - x)^{5.25}$. (This differs from the formula used in Ref. 2, but is numerically equivalent to it.) Equation 1 allows one to calculate the number of high energy muons (neglecting directly produced muons) for various muon energies, shower energies, zenith angles, and primary nuclei.

The differential primary energy spectrum for protons can be specified as $áE^{-\gamma}$. For the sake of simplicity in the present discussion assume thin the primary spectra for all nuclei have the same value of γ. We can assign weights w_i for the normalization of the different components on an energy/nucleon basis relative to protons. For example, in the model composition used by Ouldridge and Hillas, [5] the primary composition is broken into 5 groups with A = 1, 4, 14, 25, and 56 and the values of w_i are 1.0, 0.045, 0.003, 0.001, 0.00033, respectively.

For a deep underground muon detector[6] the muon energy loss rate is roughly dE/dt = a + b E, where a = 0.26 GeV hg^{-1} cm^2 and b = 4.3 X 10^{-4} hg^{-1} cm^2 and hg = hectogram = 100 grams. Integrating this energy loss over depth yields the approximate energy needed to penetrate a range R of rock.

$$E_ì = 600(e^{R/2300} \text{ hg cm-2} - 1) \text{ GeV} \qquad (2)$$

Thus, if the depth is R hg cm^{-2} of standard rock at a particular zenith angle $N_ì$ (> $E_ì$) can be evaluated for a particular detector. The number of muons fluctuates for real showers due to variations in the development of the shower and due to the random nature of the muon's survival to the depth of the detector. Luckily, however, the Monte-Carlo results indicate that the distribution of numbers of muons actually reaching a given depth is roughly a Poisson distribution, with the average value given by $N_ì$.

With the model specified above, it is possible to calculate the exclusive intensities of multiple muon events. An exclusive intensity, J_n, is defined as the number of events with exactly n muons reaching the depth of the detector in units of (time-area-solid angle)$^{-1}$. If we sum over 5 constituents of the primary composition, the result for J_n

$$J_n = \frac{\alpha \sum_{i=1}^{5} w_i A_i^n}{n!\, E_\mu^{\gamma+n-1} \cos^n\theta} \int_0^1 G^n(x)\, x^{\gamma-2}\, e^{-N_\mu}\, dx \qquad (3)$$

Notice that the factor e $^{-N1}$ in the integral depends on θ, Eì, and A_i. Thus the factors outside the integral give an exaggerated impression of the sensitivity of J_n to these variables.

For n = 1, Nì « 1 for the range of x which contributes heavily to J_1 and

$$J_1 \approx \frac{\alpha \sum w_i A_i}{E_\mu^{\gamma} \cos\theta} \int_0^1 G(x)\, x^{\gamma-2}\, dx \qquad (4)$$

In the approximation of Eq. 4, the terms outside the integral give the dependence of J1 on the primary composition, the zenith angle, and the muon energy. The sum of w_i A_i is dominated by the contribution from protons and helium nuclei for compositions resembling those measured at low energies.

It is also useful to evaluate the in for very large J_n. Crudely this is determined by the relative rates of showers with n - 1/2 < Nì < n + 1/2. For N > 10, x « 1 and G(x) = ax^{-b}, The approximate result is

$$J_n \approx \frac{\alpha}{b} E_\mu^{(1+\frac{1}{b})(1-\gamma)}\, n^{(\frac{1-\gamma-b}{b})} \sum_i w_i \left(\frac{a A_i}{\cos\theta}\right)^{\frac{\gamma-1}{b}} \qquad (5)$$

For γ = 2.8, this model gives

$$J_n \text{ æ } E_ì^{-4.18} \cos\theta^{-2.38} n^{-3.38} A_i^{2.38} \qquad \text{(for n > 10)} \qquad (6)$$

Although the single muon events were dominated by protons, the dependence of J_n for n > 10 on the atomic weight ($A_i^{2.38}$) implies that the high multiplicity events are produced predominantly by the heavier nuclei, unless heavy nuclei are very rare. For n > 10, it is seen that the exponents which control the dependence of J_n on Eì, cosθ, A_i and n are functions of b and γ only. If γ varies for different components, the result would not be as simple as outlined here. The parameter b depends on the nuclear interaction properties and is apparently sensitive to details of the interaction model. Goned[7] obtained 0.553 rather than 0.757 for the analogous parameter, although both cal cal ations used scaling models, but Goned did not use energy dependent total cross sections.

Results and Discussion. In this section I will discuss the results of the calculations of J_n for DUMAND and the usefulness of other detector systems in association with the Homestake Neutrino Detector.[8] A rough estimate of multiple muon event rates was made for DUMAND following the model outlined above. The normalization of the proton spectrum was taken from Ryan, Ormes and Balasubramanian[9] and γ was assumed to be 2.8. Table 1 shows the results of this calculation. Within the 1 km^2 area of DUMAND, showers would occur with > 1000 muons in the detector at a depth of 4-5 km. The events with 1 muon in the detector would be produced 81% of the time by proton primaries and less than 1% of the time by Fe nuclei. Events with 10 muons in the detector would be produced by hydrogen primaries about 9% of the time and by Fe nuclei about 45% of the time. It is obvious that by changing the ratio of H to Fe one can obtain drastic changes in the ratio of the rates of events with n > 10 to those with n = 1. Because of the fact that n = 1 and n > 10 come from primary particles of very different energies and the fact that the parameter γ may differ for different primary nuclei, it is not clear that sufficient information is available to determine the composition. The comparison of n = 1 events at large θ (greater depths underwater) to n > 10 events at $\theta = 0°$ could allow the composition to be studied using events from showers of similar primary energies.

TABLE 1

Exclusive Muon Event Rates

Estimated rates of multiple muon events are given for the DUMAND detector. The percentages of the events from the five primary components used in the model are also given for each multiplicity.

n	Rate	H	He	N	Mg	Fe
1	12.0 s^{-1}	81	14	2.7	1.4	0.8
2	0.28 s^{-1}	48	25	12	9	7
3	0.03 s^{-1}	24	22	18	17	19
4	0.01 s^{-1}	16	18	19	20	28
5	0.004 s^{-1}	13	16	18	21	33
6	6.6 hr^{-1}	12	14	17	21	36
7	3.6 hr^{-1}	11	14	17	21	38
8-10	4.5 hr^{-1}	11	13	16	20	40
>10	3.5 hr^{-1}	9	11	15	20	45
>100	0.4 day^{-1}	9	11	15	20	45
>1000	0.6 $year^{-1}$	9	11	15	20	45

An improvement of the composition determination could be obtained by using a Cherenkov light array or an electron array together with a deep muon detector such as DUMAND or the Homestake Neutrino Detector. Distributions of numbers of muons in the detector could be obtained for showers of comparable energy or electron size at the surface. For small showers of fixed size, the number of TeV muons would be about 8 times greater for Fe nuclei than for proton primaries. Even greater sensitivity to composition can be obtained by considering <u>inclusive</u> intensities In, defined as

$$I_n = \sum_{j=n}^{\infty} C_n^j J_j \qquad \text{where} \qquad C_n^j = \frac{j!}{n!(j-n)!} \tag{7}$$

The contributions of each primary constituent to I, are proportional to $N_{\grave{i}}^{n}$, where $N_{\grave{i}}$ is the average number of muons reaching the detector for the particular component. At fixed N_e, the factor $N_{\grave{i}}^{n}$ would be approximately 8^n times larger for primary Fe than for primary protons.[1] If I_n could be obtained for n from 1 to 5, the quantities w_i might be obtained, giving reasonably complete information about the primary composition. Details and practical shortcomings of these methods should be studied in the near future.

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