

Simulations of cosmic ray interactions: past, present, and future

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Abstract. The current status of simulations of cosmic ray interactions is reviewed. Different model approaches are compared with respect to the predicted extensive air shower characteristics. Special emphasis is put on the contribution of “hard” partonic processes whose role greatly increases with energy. Different ways to account for non-linear interaction effects are analyzed. Future prospects for model improvements and for the reduction of model uncertainties are discussed.

1. Introduction

Experimental studies of high energy cosmic rays (CR) by means of extensive air shower (EAS) techniques have already an impressive history. Since long time an essential ingredient of such activities have been simulations of CR interactions. The situation is very similar to the one at accelerators: to assure reliable measurements, one has to reach a proper level of understanding of his own experimental device. Here, an important peculiarity of EAS studies is that the atmosphere itself constitutes an important part of the “detector”. This explains the need for detailed simulations of particle propagation and interactions in air, in addition to usual “accelerator-like” modelizations of the response of ground (space-based) installations.

The backbone of a typical hadron-induced air shower is the hadronic cascade development: mean free paths and relative energy losses (inelasticities) of the primary particle and of few most energetic secondaries define the position of the shower maximum, hence also the number and lateral distribution of charged particles at ground; multiplicities of hadron-air interactions project themselves on the observed muon component. However, Monte Carlo (MC) models of hadronic interactions appear to be the weakest place in the EAS simulation chain. This is, first of all, because one can not actually describe the corresponding physics within a rigorous theoretical approach, and secondly, due to the necessity to extrapolate present phenomenological models, calibrated on the basis of available accelerator data, over many energy decades.

In past, it was customary to treat hadronic cascades in the atmosphere in a semi-inclusive way. MC procedures have been applied to determine particle free paths and inelasticities of the interactions. Then, for the given inelasticity, simple parameterizations of secondary particle spectra have been used to calculate the corresponding feeding of EAS development. Really microscopic MC models emerged comparatively recently, being requested by an increased accuracy of air shower experiments and being employed in contemporary EAS simulation programs, like CORSIKA [?] or AIRES [?].

The underlying physics picture of such models has also significantly evolved. Few decades ago it did not seem to be problematic to describe hadronic interactions at high energies: corresponding cross sections were approaching constant values with increasing energy; the multiplicity and the spectra of secondaries agreed well with the Feynman scaling picture. On the other hand, one did not feel any need to pay special attention to nucleus-nucleus interactions – as nucleus-initiated EAS were expected to be well represented by the superposition of the corresponding number of proton-induced showers. However, further experimental studies provided evidence for continuing energy-increase of cross sections, of the multiplicity of secondary particles, and of the interaction inelasticity. In turn, microscopic MC models predicted considerable fluctuations of the number of interacting projectile and target nucleons in nucleus-nucleus interactions, which give rise to large fluctuations of the EAS maximum depth, the latter being significantly greater than superposition model predictions [?, ?].

All above brings us to today’s situation: there is no simple way to extrapolate present accelerator knowledge to the highest CR energies; calculations of air shower development require sophisticated MC models for hadron-air and nucleus-air interactions. Further progress in experimental CR studies appears to be correlated with our understanding of such processes. In particular, the prospect of obtaining “model-independent” composition results may be fulfilled via thorough tests and discrimination of present MC models, using both new accelerator and CR data.

This work reviews most general features of contemporary hadronic MC generators used in CR physics, like DPMJET [?], NEXUS [?], QGSJET [?, ?], and SIBYLL [?, ?]. We discuss in a systematic way the evolution of the underlying physics picture and try to outline remaining problems and promising approaches. In particular, it is emphasized that the general scheme is rather a phenomenological one and most likely will remain so in the foreseeable future.

2. What is the basic scheme?

The general basis for contemporary models of hadronic interactions is provided by the Gribov-Regge approach [?, ?], which treats hadronic and nuclear collisions as multiple scattering processes. Each elementary re-scattering is represented at the microscopic level by a parton (quark and gluon) cascade, as depicted in figure ??(left). However, generally one can not treat such microscopic cascades explicitly, as most of those

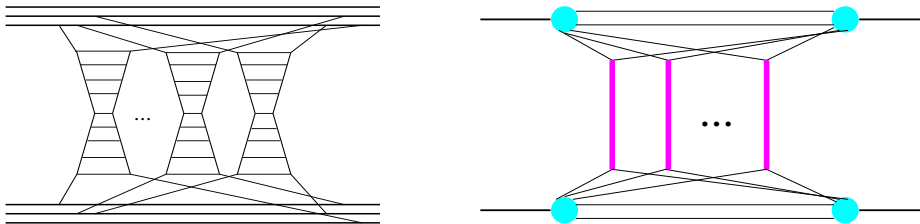


Figure 1. Typical contribution to hadron-hadron scattering amplitude. At the microscopic level, elementary re-scatterings are mediated by parton cascades (left); macroscopically they can be treated as exchanges of composite objects – Pomerons (right).

partons are characterized by small virtualities (transverse momenta), which prevents one from employing the perturbative QCD description. Instead, one can treat an elementary re-scattering process phenomenologically, as a Pomeron exchange, see figure ??(right), with the Pomeron amplitude postulated as [?, ?, ?]:

$$f_{ad}^P(s, b) = \frac{i\gamma_a\gamma_d s^{\alpha_P(0)-1}}{R_a^2 + R_d^2 + \alpha'_P(0) \ln s} \exp\left(-\frac{b^2}{4(R_a^2 + R_d^2 + \alpha'_P(0) \ln s)}\right), \quad (1)$$

where s and b are the c.m. energy squared and the impact parameter for the interaction, $\alpha_P(0)$, $\alpha'_P(0)$ are the Pomeron Regge trajectory parameters (intercept and slope) and γ_a , R_a^2 are the parameters for the Pomeron-hadron a coupling. The imaginary part of this amplitude $\chi_{ad}^P(s, b)$ is usually referred to as the Pomeron eikonal.

This allows one to calculate hadron-hadron scattering amplitude, hence, via the optical theorem, also the total cross section. Moreover, applying the Abramovskii-Gribov-Kancheli (AGK) cutting rules [?], one can separate out different contributions to the total cross section, corresponding to particular final channels of the reaction. Thus, for the inelastic cross section and for partial contributions of exactly m elementary particle production processes (“cut Pomerons”) one obtains [?, ?]:

$$\sigma_{ad}^{\text{inel}}(s) = \frac{1}{C_{ad}} \int d^2b \left[1 - e^{-2C_{ad} \chi_{ad}(s, b)}\right] \quad (2)$$

$$\sigma_{ad}^{(m)}(s) = \frac{1}{C_{ad} m!} \int d^2b \left[2C_{ad} \chi_{ad}(s, b)\right]^m e^{-2C_{ad} \chi_{ad}(s, b)}, \quad (3)$$

where the interaction eikonal χ_{ad} is defined by the Pomeron exchange, $\chi_{ad}(s, b) = \chi_{ad}^P(s, b)$, and the so-called shower enhancement coefficient C_{ad} accounts for the effects of diffraction dissociation and inelastic screening.‡ The key point of the AGK procedure is that one can neglect the interference between different configurations of inelastic interactions, i.e. between contributions with different number of elementary production processes (“cut Pomerons”). Generally, this is only valid at sufficiently high energies, where elementary parton cascades contain a large number of t -channel parton emissions, i.e. corresponding parton ladders have many rungs, see figure ??(left).

‡ Alternatively, one can treat the diffraction dissociation and inelastic screening within the Good-Walker framework [?], assuming the two hadrons to be represented by few diffractive components, each having its own coupling with the Pomeron (see [?] for a review).

To describe secondary hadron production, one assumes that elementary production processes correspond to a creation and break up of strings stretched between the projectile and the target, which results in multi-peripheral chains of secondary particles produced, – the picture being supported by the topological structure of underlying QCD diagrams [?, ?] (see also [?]). String hadronization procedures differ from model to model, with the most popular one being the LUND scheme [?], and generally involve quite a number of adjustable parameters. In the Quark-Gluon String model [?, ?] these parameters have been expressed via the intercepts of secondary Regge trajectories.

The discussed general formalism satisfies the unitarity requirements, allows to describe self-consistently the energy increase of hadronic cross sections, the violation of Feynman scaling in secondary particle spectra, the diffraction dissociation processes [?, ?, ?]. It is noteworthy that in this scheme the transition from hadron-hadron to hadron-nucleus and nucleus-nucleus collisions does not require any new parameters [?, ?]. However, the approach has no direct connection to the perturbative QCD; the high energy behavior of hadronic scattering amplitudes, hence, of the interaction cross sections is governed by the phenomenological Pomeron intercept.

3. How to match with pQCD?

To increase the predictive power of the general scheme it is very desirable to impose additional constraints on the high energy behavior of hadronic scattering amplitudes. For an elementary re-scattering process, considering again its underlying parton cascade, one finds that at sufficiently high energies a large part of the latter typically develops in the region of relatively high parton virtualities (transverse momenta) [?]. There, the smallness of the corresponding strong coupling constant $\alpha_s(p_t^2)$ is compensated by large parton multiplicity and by large logarithmic ratios of transverse and longitudinal momenta for successive parton emissions. This gives a possibility to describe such high virtuality ($|q^2| \simeq p_t^2 > Q_0^2$) parton evolution by means of pQCD techniques, with Q_0^2 being a chosen virtuality cutoff for pQCD being applicable.

In general, parton cascades contain both perturbative ($p_t^2 > Q_0^2$) and non-perturbative ($p_t^2 < Q_0^2$) parts. Applying phenomenological “soft” Pomeron treatment for the latter, these so-called semi-hard processes can be described by the contribution of “semi-hard Pomeron” exchange, the latter being represented by a piece of QCD ladder sandwiched between two Pomerons [?, ?, ?, ?], see figure ???. In an alternative scheme, the so-called mini-jet approach [?, ?, ?, ?], it is assumed that the low p_t ($p_t^2 < Q_0^2$) part of the parton cascade does not contribute significantly to secondary particle production and the semi-hard process is described by just the QCD ladder contribution. § Thus, the main difference between the semi-hard Pomeron approach, as employed in QGSJET and NEXUS, and the mini-jet scheme, which is characteristic for SIBYLL and for a number of MC models actively used in accelerator physics, e.g., PYTHIA [?, ?] and HIJING [?],

§ A formal justification for the mini-jet scheme comes from considering non-linear parton processes, as discussed in Section ??.

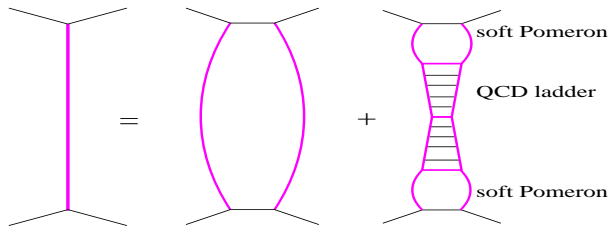


Figure 2. An elementary re-scattering process (l.h.s.) consists of the contributions of “soft” and “semi-hard” Pomeron exchanges – correspondingly the 1st and the 2nd graph in the r.h.s.

is due to an additional contribution to secondary particle production in the former case. This contribution emerges from the “soft” ($p_t^2 < Q_0^2$) parton evolution and significantly enhances the predicted inelasticity of hadronic interactions. On the other hand, both approaches are identical in what concerns high p_t hadron production. In particular, the contribution of semi-hard processes $\chi_{ad}^{\text{sh}}(s, b)$ to the hadron-hadron scattering eikonal is connected to the inclusive jet production cross section $\sigma_{ad}^{\text{jet}}(s, Q_0^2)$ (for jet transverse momenta $p_t > Q_0$):

$$2 \int d^2b \chi_{ad}^{\text{sh}}(s, b) = \sigma_{ad}^{\text{jet}}(s, Q_0^2), \quad (4)$$

where the latter is expressed via parton-parton scattering cross section $d\sigma_{ij}^{2 \rightarrow 2}/dp_t^2$ and parton momentum distribution functions (PDFs) $f_a^i(x, q^2)$ as

$$\begin{aligned} \sigma_{ad}^{\text{jet}}(s, Q_0^2) &= 2 \sum_{i,j=g,q,\bar{q}} \int dp_t^2 \int \frac{dx_i^+ dx_j^-}{x_i^+ x_j^-} \frac{d\sigma_{ij}^{2 \rightarrow 2}(x_i^+ x_j^- s, p_t^2)}{dp_t^2} \\ &\times f_a^i(x_i^+, p_t^2) f_d^j(x_j^-, p_t^2) \Theta(p_t^2 - Q_0^2). \end{aligned} \quad (5)$$

Both jet production cross section $\sigma_{ad}^{\text{jet}}(s, Q_0^2)$ and consequently the semi-hard eikonal $\chi_{ad}^{\text{sh}}(s, b)$ are strongly dependent on the choice of hadronic PDFs employed.

In addition to the semi-hard processes, at any energy one has to deal with pure soft ones, where all partons in the underlying cascade are characterized by small transverse momenta, $p_t < Q_0$. Treating the latter as usual soft Pomeron exchanges, with the eikonal χ_{ad}^{P} (see (??)), the hadron-hadron scattering eikonal χ_{ad} in (??-??) appears to be the sum of the soft and the semi-hard ones, $\chi_{ad}(s, b) = \chi_{ad}^{\text{P}}(s, b) + \chi_{ad}^{\text{sh}}(s, b)$, see figure ???. The scheme appears to be a direct generalization of the previous one, both concerning cross section calculations and for particle production treatment. The advantage is that in the very high energy limit the dominant contribution to the eikonal comes from its rapidly increasing semi-hard part and corresponding model results appear to be constrained by the pQCD predictions [?, ?, ?, ?].

Unfortunately, this scheme comes soon to its limits: the basic assumption on the individual re-scattering processes to be independent of each other is generally invalid at very high energies. By consequence, using realistic PDFs one comes to contradiction with observed hadronic cross sections and secondary particle multiplicity.

4. How to treat non-linear effects?

Considering hadronic interactions at very high energies and small impact parameters one deals with the regime of high parton densities. There, individual parton cascades are no longer independent of each other; their overlap and mutual influence give rise to a significant modification of the picture described so far. At microscopic level such non-linear parton effects are described as merging of parton ladders [?], see figure ?? . It

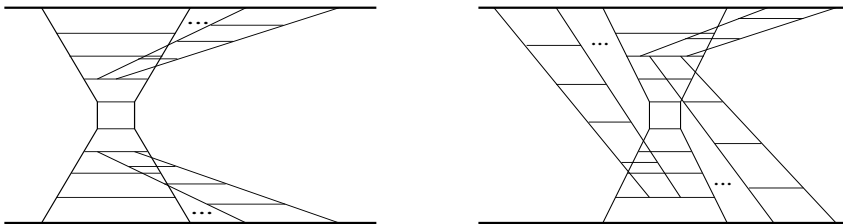


Figure 3. Examples of diagrams which give rise to non-linear parton effects.

is noteworthy that one encounters here contributions of two types. Diagrams of the kind of the one in figure ??(left) are factorizable ones: one can treat all the upper part of the corresponding graph, above the central parton box, as the contribution to the projectile parton momentum distribution; similarly the lower part defines the target PDF. Thus, adding all such contributions to the one of a simple QCD ladder, one obtains the final result in the usual QCD-factorized form (??): as a convolution of two hadron PDFs with the parton-parton scattering cross section [?]. However, such a procedure is not possible with the graph in figure ??(right): its upper and lower parts have ladder connections both to the projectile and to the target. Hence, the corresponding contribution depends on the whole collision kinematics and can not be absorbed into the corresponding PDFs. In general, due to AGK-cancellations [?], such non-factorizable graphs are not expected to influence significantly inclusive spectra of secondary particles. However, their effect on the calculated total cross sections and on the interaction dynamics is very essential [?].

A consistent treatment of non-linear interaction effects is generally very difficult. An attempt to analyze such corrections to hadronic structure functions (SFs) in the pQCD framework has been performed in [?], based on the assumption that parton densities in the low virtuality region are saturated and any further parton branchings are compensated there by the fusion of individual parton cascades. In principle, that approach may give a possibility to account for the contributions of factorizable graphs of figure ??(left), which define inclusive particle spectra. In mini-jet models one typically employs a phenomenological approach, introducing some parameterized energy-dependence of the p_t -cutoff for mini-jet production, $Q_0 = Q_0(s)$, the latter representing an effective saturation scale [?, ?]. This allows to slow down the energy increase of the total cross section and of the multiplicity of secondary particles and to reach an agreement with available data. Nevertheless, the approach has a number

of deficiencies. Most important, using an ad-hoc parameterization for the $Q_0(s)$ -dependence, one loses the connection to the pQCD treatment, which in turn leads to a low predictive power of corresponding models. Besides, one does not account for correlations of the nonlinear effects with actual parton densities at a given energy, the latter being dependent on the impact parameter for the interaction and on the projectile and target mass numbers in case of nuclear collisions. Finally, important non-factorizable contributions, arising from graphs of figure ??(right), are neglected.

An alternative approach is to treat non-linear effects within the phenomenological Gribov-Regge framework, where individual parton cascades are described by Pomeron exchanges and their mutual interactions correspond to Pomeron-Pomeron couplings [?, ?, ?]. In this scheme the multi-ladder graphs of figure ?? are replaced by so-called enhanced diagrams, which describe Pomeron-Pomeron interactions. Under the assumption that multi-Pomeron vertices are dominated by parton processes at comparatively low virtualities $|q^2| < Q_0^2$, Q_0 being a constant value, one can re-sum all significant contributions of that kind and to develop a self-consistent MC generation procedure for hadronic and nuclear collisions [?]. As a result, the approach allows one to resolve the inconsistency between realistic SFs and the observed hadronic cross sections and to account for non-linear screening effects in individual hadronic and nuclear collisions. In this scheme, the strength of non-linear effects is directly related to the corresponding parton density; the non-linear corrections are larger at small impact parameters and for bigger projectile and target mass numbers in case of hadron-nucleus and nucleus-nucleus collisions. In particular, the QGSJET-II model [?], being based on the above treatment, appears to be in agreement with the data of the RHIC collider on the multiplicity of secondary hadrons produced in central nucleus-nucleus interactions. However, by construction the scheme treats non-linear effects as arising from low p_t ($p_t^2 < Q_0^2$) parton processes. In the region of very high parton densities one may expect that parton density saturates at the Q_0 -scale and further corrections arise from perturbative ($|q^2| > Q_0^2$) multi-ladder couplings. Such effects, being the main subject of investigation in the GLR approach [?], are missing in the described scheme. In general, one could obtain an additional suppression of secondary particle production, when corresponding corrections are taken into account.

5. How certain are model predictions for air showers?

It is generally difficult to establish a direct connection between individual hadronic model constructions and the predicted EAS characteristics. Various model features influence air shower development in a rather non-trivial and sometimes unexpected way. Thus, it is useful to introduce an intermediate step, comparing model predictions for hadron-air interactions at the macroscopic level, primarily, concerning inelastic cross sections, inelasticity (defined below as the energy fraction of the fastest secondary nucleon (charged meson) for $p - ^{14}N$ ($\pi - ^{14}N$) collisions), and multiplicity.

In figures ??,?? these characteristics are plotted for QGSJET [?], QGSJET-II [?],

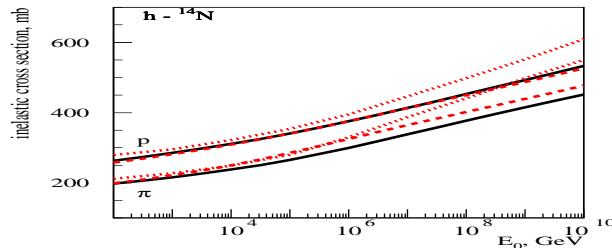


Figure 4. Inelastic hadron-nitrogen cross sections for QGSJET, QGSJET-II, and SIBYLL 2.1 – full, dashed, and dotted curves correspondingly.

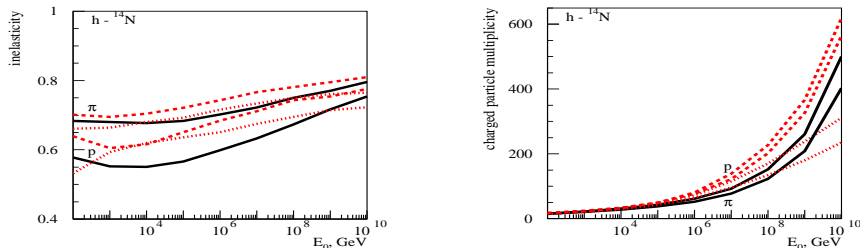


Figure 5. Inelasticity (left) and multiplicity of charged particles (right) in $p - {}^{14}\text{N}$ and $\pi - {}^{14}\text{N}$ collisions as calculated with the QGSJET, QGSJET-II, and SIBYLL 2.1 models – full, dashed, and dotted curves correspondingly.

and SIBYLL 2.1 [?]. Most essential differences between these models arise from the treatment of semi-hard processes, which depends on the chosen PDFs, and from the implementation of non-linear screening effects. The latter are not taken into account in the original QGSJET, which is based on the linear quasi-eikonal Gribov-Regge scheme and the semi-hard Pomeron approach. Correspondingly, that model uses rather “flat” (pre-HERA) parton distributions, not consistent with present measurements. SIBYLL, being a mini-jet type model, is based on the GRV94 PDFs [?], which are consistent with HERA data, and introduces non-linear effects via an energy-dependent Q_0 -cutoff. Finally, in QGSJET-II parton distributions are calculated in the semi-hard Pomeron framework and fitted to HERA data; non-linear effects are treated within the above-described approach. In the high parton density regime such effects lead to a considerable suppression of “soft” (low p_t) hadron production and the model approaches the mini-jet limit. Although the PDFs in QGSJET-II increase much faster in the small Feynman x limit than in original QGSJET, being now fixed by the measured structure function $F_2(x, Q^2)$, the corresponding effect is essentially compensated by non-linear screening corrections in what concerns proton-air cross section. The corresponding energy increase is significantly slower than in the SIBYLL model, where non-linear effects are introduced via a p_t -cutoff for semi-hard processes and thus are neglected for non-perturbative (low p_t) ones. On the other hand, the influence of non-linear effects on the particle production in hadron-nucleus and nucleus-nucleus collisions is much stronger compared to hadron-hadron case. As a consequence, the inelasticity and multiplicity of hadron-air collisions are significantly reduced in QGSJET-II, compared to QGSJET results, being in a wide energy range even lower than in the SIBYLL model, see figure ??.

The discussed differences in the secondary particle production make a strong impact on the calculated EAS characteristics. In QGSJET-II, the position of the shower maximum X_{\max} is systematically shifted deeper in the atmosphere, compared to QGSJET, as seen in figure ??(left), being now close to the SIBYLL results. On

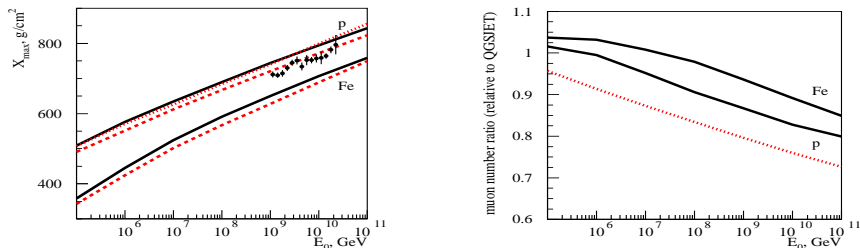


Figure 6. Left: shower maximum depth for proton- and iron-induced EAS, as calculated with QGSJET, QGSJET-II, and SIBYLL – full, dashed, and dotted curves correspondingly, compared to HIRES data [?]. Right: EAS muon number ($E_{\mu} > 1$ GeV) at sea level for proton- and iron-induced vertical EAS, as calculated with the QGSJET-II and SIBYLL models, relative to QGSJET results.

the other hand, one obtains a sizable reduction of EAS muon number, as shown in figure ??(right), with the difference between the QGSJET-II and SIBYLL models being only about 10% at highest CR energies.

Thus, with the non-linear screening corrections taken into account, the EAS predictions of contemporary hadronic interaction models appear to be rather close to each other. Still, at the particle production level the models give significantly different results. This is both due to the above-discussed differences in the basic physics picture, and due to the impossibility to fix model parameters and algorithms in a unique way, based on available data. For example, choosing a different parameterization for the energy dependence of the p_t -cutoff $Q_0(s)$ in SIBYLL, one can significantly modify the results obtained [?, ?]. Another serious uncertainty in model constructions is connected to the treatment of leading particle production, including the effects of energy-momentum sharing between elementary multiple scattering processes [?, ?], the behavior of hadronic leading states [?], etc. Moreover, one can not expect a significant improvement of the situation in a foreseeable future, as the corresponding treatment will essentially remain a non-perturbative one. As an illustration, one can investigate the sensitivity of the results to the choice of the momentum distribution of constituent partons (string “ends”), involved in elementary re-scattering processes. Changing from the default “valence-like” distribution of QGSJET, which corresponds at $x \rightarrow 0$ to $x^{-0.5}$ behavior [?], to a “softer” $x^{-0.7}$ law, one can obtain equally good description of available collider data. On the other hand, the latter case corresponds to a smaller inelasticity and multiplicity of hadron-air collisions and, consequently, to a deeper air shower penetration and smaller EAS muon number, as shown in figure ??.

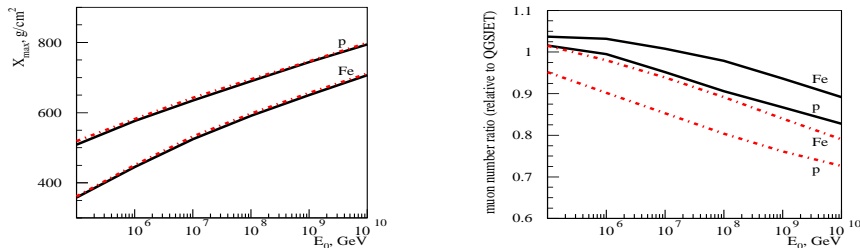


Figure 7. The same as in figure ??, calculated with the default QGSJET-II and using modified string end distributions (as explained in the text) – full and dot-dashed curves correspondingly.

6. What to expect in future?

As demonstrated above, there is a considerable similarity in basic constructions of contemporary hadronic interaction models. Both due to the progress in model development and due to an increasing amount of available accelerator data the models converge in their predictions for EAS characteristics. Still, comparing all MC generators available at the market, the spread in the corresponding predictions remains rather large [?]. Partly, this is due to the discussed differences in the basic algorithms employed. However, even within a particular model approach one can not presently fix in a unique way the parameters and technical algorithms, which is both due to the lack of sufficient experimental information and because the underlying theoretical picture is not yet powerful enough to provide more strict constraints on model extrapolations towards very high energies.

In future one may expect significant progress in low- x QCD studies, in particular, in the BFKL framework [?, ?]. Correspondingly, present phenomenological approaches to the treatment of semi-hard processes, like the mini-jet and semi-hard Pomeron schemes, could be replaced by a rigorous QCD treatment. Still, non-linear interaction effects remain an important issue. While for the “soft” (low p_t) processes the problem is essentially solved at a phenomenological level [?], the perturbative description of these phenomena in the high p_t region remains a challenge. Present theoretical approaches, e.g., ones based on the semi-classical QCD scheme [?], are far from being applicable for general hadron-air and nucleus-air collisions. However, in a remote future we may expect a consistent scheme to emerge. Moreover, at sufficiently high energies the bulk of the interactions is expected to be dominated by the perturbative high p_t processes and the corresponding treatment could be a rather unambiguous one.

Still, it does not mean that future models will provide strict theoretical results for the spectra of secondary particles in general minimum-bias collisions. Even in a “perfect” model, after having determined the general configuration of an inelastic interaction and having processed perturbative parton cascading, one has to treat the hadronization process – the conversion of final partons into observed hadrons. Besides, as already mentioned above, a considerable uncertainty comes from the treatment of hadronic leading states. Thus, we have to live with the fact that hadronic MC

models will remain to a large extent phenomenological ones and will have some inherent uncertainties. There is a great potential to reduce the latter with forthcoming data of RHIC and LHC colliders, in particular, if the experiments will provide additional information on leading particle spectra. On the other hand, the increasing accuracy of air shower experiments already now allows to test model algorithms in an indirect way and to discriminate between available models [?, ?, ?, ?].

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