Groping for the (cosmic-ray) knee

Frank Jones, Robert Streitmatter, and Demos Kazanas

Exploring the Universe Division, NASA/Goddard Space Flight Center, Greenbelt, MD, USA

Abstract. We consider the possibility that the cosmic-ray knee is produced by a change in the nature of the hadronic interaction at approximately 1 TeV center of mass. Specifically we explore the result of the opening up of a new production channel whose output is undetectable by current air shower techniques.

1. The problems

One current explanation of the cosmic-ray knee asserts that the spectrum of supernova produced cosmic rays ends at approximately 3×10^{15}eV and is replaced, above this energy by a spectrum of particles from a different, perhaps extra-galactic, source. The problem with this particular explanation is that it requires a rather precise matching of the two spectra in two characteristics that should have no particular relationship with each other.

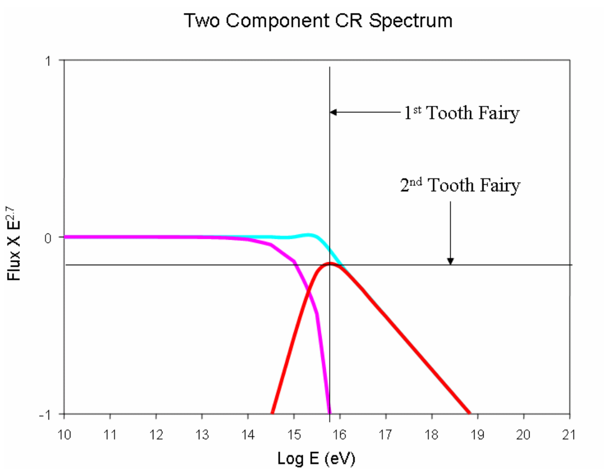

Figure 1. If the knee is attributed to two separate components their magnitudes AND energy scales must match precisely.

As we see in Figure **??** the energy at which one spectrum leaves off and the other begins must match rather precisely and so should their relative normalizations in order to produce a smooth total spectrum. This is sometimes called a “Two Tooth Fairy” problem.

Another difficulty arises when one observes the data concerning the chemical composition of the cosmic-ray beam as energy approaches and passes through the knee. All of the data taken collectively gives no particular trend except to begin to diverge widely as the knee is approached from below and beyond (Figure **??**). If, however, one takes a subset of the data as shown in Figure **??**, a trend towards proton-like composition appears as the knee is approached from below only, to change to a definite heavier trend as soon as the knee is passed.

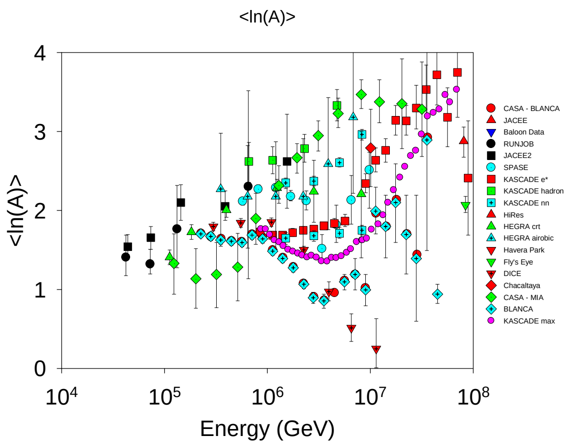

Figure 2. No theory can explain this spread of results.

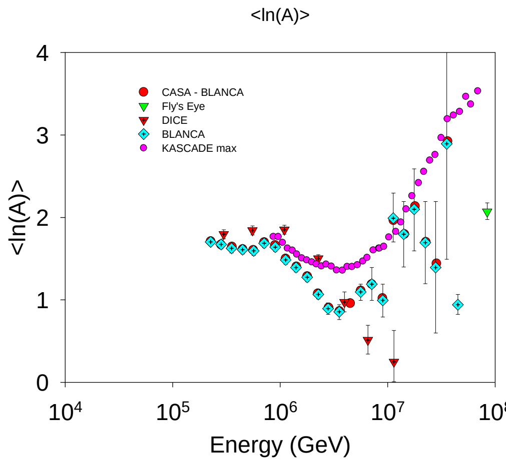

Figure 3. Keep only that data that is at least somewhat consistent.

2. Model assumption 1.

We considered that, first of all, if the trend to proton-like composition with increasing energy below the knee is correct it can only mean that the heavies are beginning to drop out with increasing energy. But, if this is what is occuring the protons should be dropping off at an even lower energy because almost all acceleration models have the produced spectrum end at a particular rigidity or energy per unit charge so that particles with higher charge would cut off at a higher energy. Therefore, any protons accelerated by the same process that accelerates the heavies could not account for the reduction in $< \log(A) >$ so there must be an additional source of protons that extends to much higher energy. Our first assumption will be that there is an additional, somewhat harder, spectrum of protons in the cosmic-ray beam of an, admittedly, unknown origin. In Figure **??** we show this additional proton spectrum and its effect on the total spectrum. We show in Figure **??** the effect on $< \log(A) >$ of this assumed additional proton source.

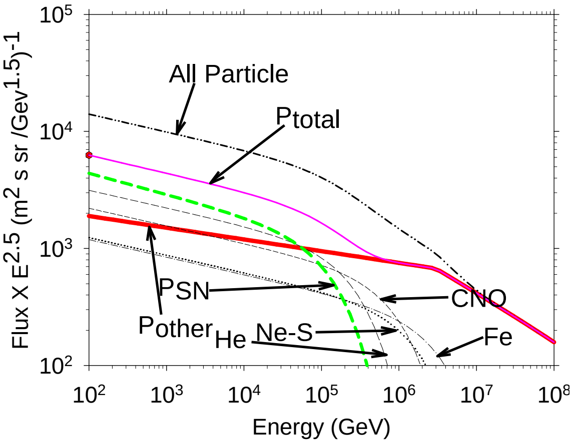

Figure 4. Assume an additional, somewhat harder, spectrum of pure proton composition.

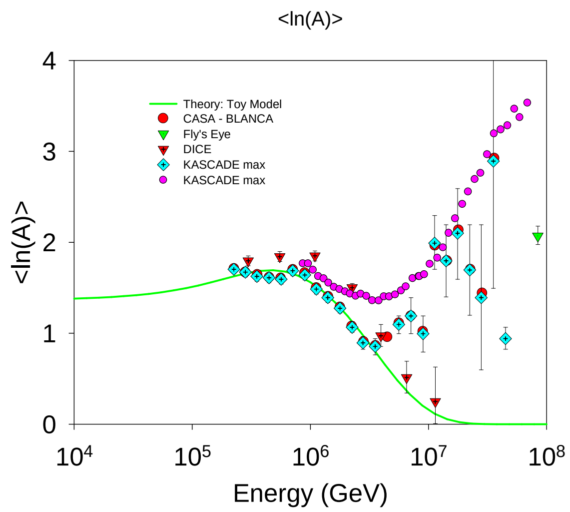

Figure 5. The additional proton spectrum can produce the drop off of $< \ln(A) >$ below the knee but can not reproduce the trend to heavier primaries above the knee.

3. Model assumption 2.

It should be pointed out that we also have not produced a bend in the spectrum hence we, as yet, have no knee.

To proceed we recall the history of how the knee in the cosmic-ray spectrum was discovered. In 1956 Vernov *et al.*[?] pointed out that a change in the nature of the interaction between particles at ultra high energy could produce an "irregularity" in the size spectrum of cosmic-ray induced air showers. Two years later, in 1958, Kulikov and Khristiansen reported [?] the observation of an irregularity that we now call the knee of the spectrum.

At this point we make an assumption about the interaction cross section between the high energy protons and the air nuclei that produce the shower cascades. First of all we assume that, at a critical energy E_c, a new production channel opens up in the particle collision cross section and that the products of this channel are invisible, resulting in lost energy. This produces an apparent steepening of the cosmic-ray spectrum because more energy is lost in the shower cascade as the primary energy increases due to the increase in the number of interaction generations occuring above the critical energy. We follow the approach of Kazanas and Nicolaidis [**?**]

To examine the effect of this assumption on the development of an air shower we employ the "Toy Model" devised by Heisenberg to describe electromagnetic cascades and clearly elucidated by Gaisser.

4. The toy model

Consider two types of products from a hadron collision, "A" type particles that interact (more hadrons) and "B" type particles that don't (carry away "lost" energy). In each collision there are α A type particles and β B type particles produced as long as the energy of the incoming particle is above the critical energy E_c. Further, consider that in each collision all of the incoming energy is shared equally among the $\alpha + \beta$ produced particles. The energy going into B type particles is lost so after the first collision the lost energy is

$$E_{L1} = \frac{E_0 \beta}{\alpha + \beta}. \tag{1}$$

The energy remaining for the second interaction level in the chain is

$$E_{R1} = \frac{E_0 \alpha}{\alpha + \beta}. \tag{2}$$

The energy lost in the second collision level in the chain is given by

$$E_{L2} = \frac{E_{R1} \beta}{\alpha + \beta} = \frac{E_0 \alpha \beta}{(\alpha + \beta)^2} \tag{3}$$

and for the n^{th} collision level in the chain we have

$$E_{Ln} = \frac{E_0 \alpha^{n-1} \beta}{(\alpha + \beta)^n} \tag{4}$$

and the energy remaining in the collision chain is just

$$E_{Rn} = E_0 \left(\frac{\alpha}{\alpha + \beta} \right)^n. \tag{5}$$

After N collision levels the total amount of energy lost to the process is just the sum of the energy lost at each level

$$E_{LN} = \sum_{n=1}^{N} E_{Ln} = E_0 \frac{\beta}{\alpha} \sum_{n=1}^{N} \left(\frac{\alpha}{\alpha + \beta} \right)^n \tag{6}$$

$$= \frac{E_0 \beta}{\alpha + \beta} \left(\frac{\left(\frac{\alpha}{\alpha+\beta} \right)^N - 1}{\left(\frac{\alpha}{\alpha+\beta} \right) - 1} \right) \tag{7}$$

$$= E_0 \left(1 - \left(\frac{\alpha}{\alpha+\beta}\right)^N\right) \tag{8}$$

$$= E_o - E_{RN} \tag{9}$$

Let there be a critical energy below which a particle can not produce a B type particle. Then we have

$$E_c = \frac{E_{RN_c}}{\alpha^{N_c}} = \frac{E_0}{(\alpha+\beta)^{N_c}} \tag{10}$$

and

$$N_c = \frac{\ln\left(\frac{E_0}{E_c}\right)}{\ln(\alpha+\beta)} \tag{11}$$

and the total energy lost in the shower is

$$E_{LT} = E_0 \left(1 - \left(\frac{\alpha}{\alpha+\beta}\right)^{N_c}\right) \tag{12}$$

If the CR flux is really

$$j(E) = \text{const.} \left(\frac{E}{E_c}\right)^\gamma \tag{13}$$

but due to energy loss appears to be

$$j'(E') = \text{const.} \left(\frac{E'}{E_c}\right)^\Gamma \tag{14}$$

then we have

$$\left(\frac{E'}{E_c}\right) = \left(\frac{E}{E_c}\right)^{\gamma-1/\Gamma-1} \tag{15}$$

But we have (identifying $E' \Longleftrightarrow E_{R_{N_c}}$, $E \Longleftrightarrow E_0$)

$$\left(\frac{E'}{E_c}\right) = \left(\frac{E}{E_c}\right)\left(\frac{\alpha}{\alpha+\beta}\right)^{N_c(E)} \tag{16}$$

and

$$\left(\frac{E}{E_c}\right)^{\frac{\gamma-1}{\Gamma-1}-1} = \left(\frac{\alpha}{\alpha+\beta}\right)^{N_c(E)}. \tag{17}$$

Inserting the formula

$$N_c = \frac{\ln\left(\frac{E_0}{E_c}\right)}{\ln(\alpha+\beta)} \tag{18}$$

we have

$$\left(\frac{E}{E_c}\right)^{\frac{\gamma-1}{\Gamma-1}-1} = \left(\frac{E}{E_c}\right)^{\frac{\ln\left(\frac{\alpha}{\alpha+\beta}\right)}{\ln(\alpha+\beta)}} \tag{19}$$

and

$$\frac{\gamma-1}{\Gamma-1} - 1 = \frac{\ln\left(\frac{\alpha}{\alpha+\beta}\right)}{\ln(\alpha+\beta)} \tag{20}$$

$$\frac{\gamma - 1}{\Gamma - 1} = \frac{\ln(\alpha)}{\ln(\alpha + \beta)} \approx 0.85 \tag{21}$$

$$\beta = \alpha^{1/.85} - \alpha \tag{22}$$

Figure **??** shows the apparent spectrum that is produced by introducing the new "invisible" component produced in any collision with incoming energy greater than E_c. The bump in the spectrum is produced by smoothing in the graph the discontinuity produced by the abrupt opening of the new channel. In reality one would expect the threshold to be more gradual producing a smaller or non-existent bump.

We have not yet produced the trend toward heavier composition that is seen above the knee. Since we have postulated that the cosmic-ray beam consists of pure protons at this energy the trend to heavier nuclei must be only apparent. This can be produced by assuming that the cross section for hadron production undergoes a threshold effect at E_c and increases enough to raise the height of maximum development of the shower giving the appearance of being initiated by a heavier particle.

5. Model assumption 3

If the total cross section increases for energies $> E_C$ then each leg of the cascade in this energy region will be shorter by an amount ϵ. Since the number of such legs is just N_c the total amount the shower maximum will be raised is $N_c\epsilon$.

For a normal shower with $A = 1$ X_{max} is given by

$$X_{max} = \lambda \frac{\ln(E/E_k)}{\ln(\alpha)} \tag{23}$$

where E_k is the energy below which the shower stops growing (no more secondary

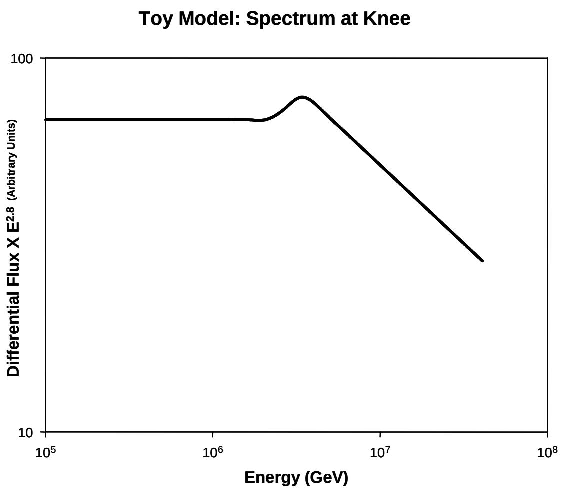

Figure 6. Apparant spectrum from theory including "lost" energy

particles are emitted in a collision). Since we have

$$N_c = \frac{\ln(E/E_c)}{\ln(\alpha+\beta)} \tag{24}$$

and so

$$X_{max} = \lambda\frac{\ln(E/E_k)}{\ln(\alpha)} - \epsilon\frac{\ln(E/E_c)}{\ln(\alpha+\beta)} \tag{25}$$

For a nucleus with $A \neq 1$ we have

$$\begin{aligned} X_{max} &= \lambda\frac{\ln(E/AE_k)}{\ln(\alpha)} \\ &= \lambda\frac{\ln(E/E_k)}{\ln(\alpha)} - \lambda\frac{\ln(A)}{\ln(\alpha)} \end{aligned} \tag{26}$$

since it develops like A showers of initial energy E/A in the superposition model. Combining Equations (**??**) and (**??**) we obtain

$$\lambda\frac{\ln(A)}{\ln(\alpha)} = \epsilon\frac{\ln(E/E_c)}{\ln(\alpha+\beta)} \tag{27}$$

and so the apparent A in a shortened shower is given by

$$A = \left(\frac{E}{E_c}\right)^{\left[\frac{\epsilon}{\lambda}\frac{\ln(\alpha)}{\ln(\alpha+\beta)}\right]} \tag{28}$$

But we have from Equations (**??**) and (**??**)we have

$$\left(\frac{E}{E_c}\right)^{\left[\frac{\ln(\alpha)}{\ln(\alpha+\beta)}\right]} = \left(\frac{E'}{E_c}\right). \tag{29}$$

So in terms of the apparent energy of the shower we have

$$A = \left(\frac{E'}{E_c}\right)^{\epsilon/\lambda} \tag{30}$$

We can see in Figure **??** that an increase in the hadron production cross section at E_c can produce an apparent trend to heavier primaries above the knee. Unfortunately, we see from Figure **??** that this effect has a tendancy to run away. In principle the effect could be turned off at an energy above E_c but this could only make the elongation curve begin to run parallel to the proton/lead curves and would not return it to the proton curve as the data from HiRes [**?**] indicates.

This indicates to us that theories of this type (change in the fundamental interaction) to explain the phenomena associated with the knee of the cosmic-ray spectrum face serious difficulties in explaining all of the (now) well established data.

[1] Vernov, Zatsepin, Khristiansen and Chudakov 1956 *Report to the All Union Conference on Cosmic Rays*, Tbilisi

[2] Kulikov and Khristiansen 1958 *JETP* **35** 635

[3] Kazanas D and Nicolaidis A 2003 *General Relativity and Gravitation* **35** 1117

[4] Heitler W 1944 *Quantum Theory of Radiation* (Oxford: Oxford University Press)

[5] Gaisser T K 1990 *Cosmic Rays and Particle Physics* (Cambridge: Cambridge University Press)

[6] Abbasi R U, et al. (The High resolution Fly's Eye Collaboration 2005 *ApJ* **622** 910

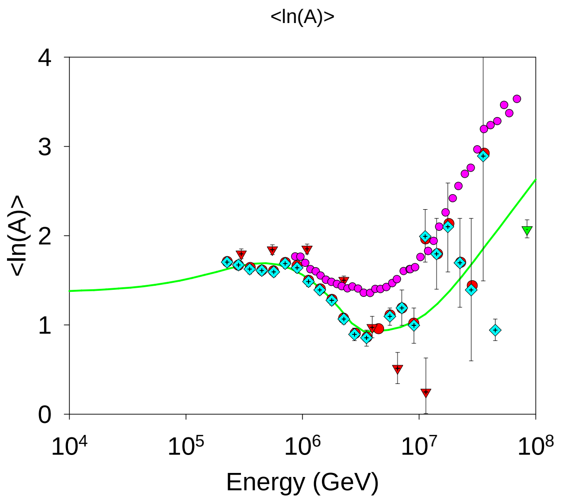

Figure 7. An increase in the hadron production cross section at E_c can produce an apparent trend to heavier primaries above the knee.

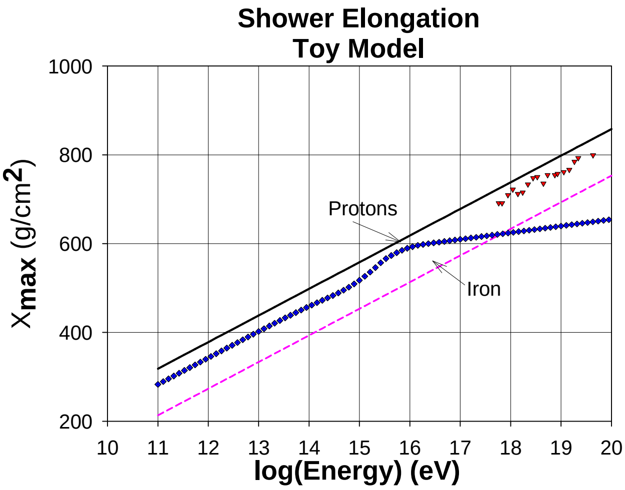

Figure 8. The model has a tendancy to run away as the energy increases.